TOKYO J. MATH.
VOL. 1, NO. 1, 1978

On the Values of Eisenstein Series

Dedicated to Professor Yukiyoshi Kawada on his 60th Birthday

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Introduction.

The main purpose of the present paper is to settle the theorem of v. Staudt-Clausen for 'normalized Hurwitz-Herglotz function' $H_s(\tau; u, v)$ in the singular case (i.e. the case τ is imaginary quadratic and $u, v \in \boldsymbol{Q}$):

$$H_s(\tau; u, v)=\frac{s!}{12\sqrt{\Delta^s}}\sum_{(m,n)\in Z^2}{}' \frac{e^{2\pi i(mu+nv)}}{(m\omega_1+n\omega_2)^s}$$

where $\tau=\omega_2/\omega_1$ and $\Delta=\Delta(\omega_1, \omega_2)$ is the usual discriminant function for Weierstrass' $\wp$-function with periods ω_1, ω_2.

The result is, roughly speaking, that the 'theorem of v. Staudt-Clausen' is of the same type as Herglotz except for an algebraic additive term whose denominator is divisible by at most prime factors of a finite number of integers given in the respective case.

Here note that in $\boldsymbol{Q}(\sqrt{-1})$, for example, $H_s(\sqrt{-1}; u, v)$ does not vanish and has an additive contribution mentioned above to v. Staudt-Clausen even for $s\not\equiv 0 \pmod 4$, while $H_s(\sqrt{-1}; 0, 0)$, the Hurwitz-Herglotz number, vanishes for $s\not\equiv 0 \pmod 4$.

Further it should be noted that as a byproduct of our theory, an interesting identity is obtained from modular transformation formula for function W_λ (see **2.2**).

In the final part, we add some comment on Ramanujan's formula for series of Lambert type.

§ 1. Kronecker's function K.

1.1. Let w, τ be complex variables and Im τ be positive. We define

$$\vartheta_1(w, \tau)=\sum_{n=-\infty}^{\infty} e^{\pi i(n+1/2)^2\tau+2\pi i(n+1/2)(w-1/2)} \tag{1.1}$$

Received January 14, 1978