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Some Criteria for the First Case of Fermat's Last Theorem

Dedicated to the memory of my friend Professor Masao NARITA

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Introduction.

Let p be a prime. We say that the first case of Fermat's last theorem (FLT) fails for the exponent p when there exist integers x, y, z, such that $p\nmid xyz$ and $x^p+y^p+z^p=0$. We shall indicate some congruences which follow from the assumption that the FLT fails for p. These will involve the Bernoulli polynomials

$$B_n(X)=\sum_{j=0}^{n}\binom{n}{j}B_jX^{n-j}\quad(\text{for } n\geqq 0)\ .$$

We require also some values of the Bernoulli polynomials and its functional equation:

$$B_n(1-X)=(-1)^nB_n(X),\ \text{and for } n \text{ even:}$$

$$B_n\left(\frac{1}{3}\right)=B_n\left(\frac{2}{3}\right)=\frac{(1-3^{n-1})B_n}{2\times 3^{n-1}}\ ,$$

$$B_n\left(\frac{1}{6}\right)=B_n\left(\frac{5}{6}\right)=\frac{(1-2^{n-1})(1-3^{n-1})B_n}{2^n\times 3^{n-1}}\ .$$

E. Lehmer proved in [1] (1938), the following congruences (for $2\leqq n<p$):

$$\sum_{j=1}^{[p/n]}(p-jn)^{2k}\equiv\frac{n^{2k}}{2k+1}\left\{\frac{2k+1}{n}pB_{2k}-B_{2k+1}\left(\frac{s}{n}\right)\right\}\quad(\text{mod } p^3)\ ,$$

$$\sum_{j=1}^{[p/n]}j^{2k-1}\equiv\frac{1}{2k}\left\{B_{2k}\left(\frac{s}{n}\right)-B_{2k}\right\}-\frac{p}{n}B_{2k-1}\left(\frac{s}{n}\right)\quad(\text{mod } p^2)\ ,$$

where $p\equiv s\ (\text{mod } n)$, $1\leqq s\leqq n-1$. Schwindt proved in [4] (1933):

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