

On Singularities of the Harish-Chandra Expansion of the Eisenstein Integral on Spin(4,1)

Masaichi MAMIUDA

Waseda University

Introduction.

Let G be the universal covering group of De Sitter group and $G=KAN$ be its Iwasawa decomposition. Let MAN be a minimal parabolic subgroup of G . We consider the Eisenstein integral for MAN which is defined as follows;

$$(1) \quad E(s, v, x) = \int_K e^{(s-3/2)t(xk)} \tau_1(k(xk)) v \tau_2(k^{-1}) dk, \quad \text{for } s \in \mathbb{C}, x \in G,$$

where τ_i are irreducible unitary representations of K on V_i ($i=1, 2$) and $v \in \mathcal{V}_M = \{v; \text{ a linear endomorphism of } V_2 \text{ into } V_1 \text{ with } \tau_1(m)v = v\tau_2(m) \text{ for all } m \in M\}$ (see, §3, [3], [5], [15]). Then Eisenstein integral $E(s, v, a_t)$ has the series expansion on a Weyl chamber A^+ , which is divided into two parts associated with the Weyl group W of the pair (G, A) ;

$$(2) \quad E(s, v, a_t) = E(s, t)C_1(s)v + E(-s, t)C_w(s)v, \quad \text{for } t > 0,$$

where $E(s, t) = e^{(s-3/2)t} \sum_{k=0}^{\infty} A_k(s) e^{-kt}$. (See, Harish-Chandra [3].) The eigenvalues of Casimir operator on G is parametrized by (s, v) and the function $x \mapsto E(s, v, x)$ is an eigenfunction corresponding to (s, v) . By the change of variable $y = e^{-t}$, the differential equation for $E(s, v, a_t)$ is transformed to an ordinary differential equation with a regular singular point at $y=0$. The formula (2) gives the series expansion of the solution $E(s, v, a_t)$ around the regular singular point $y=0$. As the classical theory of differential equations teaches us, the coefficients $A_k(s)$ ($k \in \mathbb{Z}$) satisfies a certain recurrence formula with respect to k and they are not well-defined for all $s \in \mathbb{C}$. So, the formula (2) is valid on a certain open dense connected subset $\mathcal{O}(\tau_1, \tau_2)$ of \mathbb{C} . But the function $s \mapsto E(s, t)$ can be extended to a meromorphic function on \mathbb{C} with values in the space of all linear endomorphisms on \mathcal{V}_M (see, §3). In this paper, we shall say