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On the First Cohomology Group of a Minimal Set

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Introduction.

It is one of the most important problems in the theory of topological dynamics to determine what space can be a minimal set under a continuous flow. For example, it has been conjectured that there is no minimal flow on the 3-sphere S^3. In this paper, we shall study the first cohomology of minimal sets.

It is known that the space on which an almost periodic minimal flow or a distal minimal flow exists has a non-trivial first cohomology group. However the "almost periodicity" and the "distality" are both destroyed by a time-change, while the "minimality" is invariant by a time-change. The method for calculating the first cohomology of minial sets which is exhibited in this paper is quite independent of the parametrization by the time.

In § 3 we will establish a method for calculating the first cohomology of a minimal set from certain 0-th cohomology groups. As an application of the consequence of § 3, we can get a method for deciding the first cohomology of a minimal set which forms a 3-dimensional manifold (§ 4, Theorems 1 and 2). And in § 5 we will investigate on 1-cycles of a 3-dimensional minimal set. § 1 and § 2 are preliminaries. Higher dimensional cases can be treated by the same way, but it seems to be impossible to prove the non-triviality of the first cohomology of a minimal set by our method in the case of higher dimensional manifolds. Hence we do not treat the higher dimensional case in this paper. In the case of 3-manifolds, our method seems to be useful for the proof of the non-triviality of the first cohomology of a minimal set.

In the case when the minimal set is a two dimensional manifold, using our method, we can decide the first cohomology of it completely. But it is well-known that the only two dimensional manifold admitting a minimal flow on it is the 2-torus. Therefore the results for two

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