TOKYO J. MATH.
VOL. 1, NO. 1, 1978

Geometry on Complements of Lines in P^2

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(Communicated by K. Kodaira)

Introduction.

Let $\Delta_0, \cdots, \Delta_q$ be projective lines on a complex projective plane P^2, where $\Delta_i \neq \Delta_j$ for $i \neq j$. We shall study algebro-geometric properties of the complement $S = P^2 - \bigcup \Delta_j$. For instance, we shall compute the logarithmic geometric genus $\bar{p}_g$, logarithmic irregularity $\bar{q}$, logarithmic m-genus $\bar{P}_m$, logarithmic Kodaira dimension $\bar{\kappa}$, logarithmic Chern numbers $\bar{c}_1^2$, $\bar{c}_2$ of S and establish fundamental relations among them. For the definitions of $\bar{p}_g$, $\bar{q}$, $\bar{P}_m$, $\bar{\kappa}$ we refer the reader to [4] and [5].

THEOREM I. *$\bar{q} = q$ holds. If $q \geqq 2$ and $\bar{p}_g < q-1$, then $S = C \times \Gamma$, $\bar{p}_g = \bar{P}_m = 0$ for any $m \geqq 1$; $\bar{\kappa} = -\infty$, $\bar{c}_1^2 = 3-2q$, $\bar{c}_2 = 1-q$. If $\bar{p}_g = 1$, $q = 2$, then $S = C^{*2}$, $\bar{\kappa} = 0$ and $\bar{c}_1^2 = \bar{c}_2 = 0$. If $\bar{p}_g = q-1 \geqq 2$, then $S = C^* \times \Gamma$, $\bar{g}(\Gamma) \geqq 2$ and $\bar{\kappa}(S) = \bar{\kappa}(\Gamma) = 1$; $\bar{c}_1^2 = \bar{c}_2 = 0$. Finally, if $\bar{p}_g \geqq q$, then $\bar{p}_g \geqq 2q-4$, $\bar{\kappa} = 2$ and $5\bar{c}_2 \geqq 2\bar{c}_1^2$.*

Summarizing the results, we obtain the following

TABLE

Type of Δ	$\bar{\kappa}$	$\bar{q}=q$	$1-\bar{q}+\bar{p}_g$	$\bar{c}_1^2$	$\bar{c}_2$	S
I	$-\infty$	0	1	4	1	C^2
		1	0	0	0	$C \times \Gamma$
		$\geqq 2$	$1-\bar{q}$	$3-2q$	$1-q$	$\bar{g}(\Gamma) = q \geqq 1$
II	0	2	0	0	0	$C^* \times C^*$
II$^1/_2$	1	$\geqq 3$	0	0	0	$C^* \times \Gamma, \bar{g}(\Gamma) = q-1$
III	2	$\geqq 3$	$\geqq 1$ $\geqq \bar{q}-3$	$\geqq 1$	$\geqq \frac{2}{5}\bar{c}_1^2$	

Here, type of Δ is defined as follows:

Received January 1, 1978