Tokyo J. Math.
Vol. 1, No. 2, 1978

On Lattice Isomorphisms of $C(X)^+$

Isaac NAMIOKA and Sadahiro SAEKI

University of Washington and Tokyo Metropolitan University

Introduction.

This paper originates from a question raised by Professor J. J. Schäffer during the meeting on Banach spaces at Kent State University in August, 1977. The question is this: Let X and Y be compact Hausdorff spaces, let $C(X)^+$(resp. $C(Y)^+$) be the lattice of non-negative continuous functions on X(resp. Y), and let T be a lattice isomorphism of $C(X)^+$ onto $C(Y)^+$; does T preserve the strict inequality $<$? Here, for f and g in $C(X)^+$, $f<g$ means that $f(x)<g(x)$ for each x in X. By Kaplansky's theorem [7], if $C(X)^+$ and $C(Y)^+$ are lattice isomorphic, then X and Y are homeomorphic, and so we may assume that $X=Y$. It turns out that the answer to Schäffer's question depends on the space X, and the rather unexpected result is: Each lattice isomorphism of $C(X)^+$ onto itself preserves the strict inequality if and only if X is not the Stone-Čech compactification of a non-compact, σ-compact, locally compact Hausdorff space. If X satisfies this condition, we say that the space X has property (S). (The reason for our choice of the letter "S" should, by now, be clear.) Professor E. Hewitt then started to ask us questions concerning the case where X and Y are not assumed to be compact. Then we can no longer assume that $X=Y$, and the answer to Schäffer's question (as generalized by Hewitt) depends on the topological properties of X and Y. Property (S), suitably generalized, again plays the central role. The purpose of the present paper is to present the answers to the questions of Schäffer and Hewitt, to investigate related questions, and to establish further properties of spaces with (S).

The paper is organized as follows: Section 1 contains characterizations of those compact Hausdorff spaces X such that each lattice isomorphism $C(X)^+\rightarrow C(X)^+$ preserves the strict inequality. The proofs are relatively simple and transparent.

Section 2 contains generalizations of the results of Section 1 to non-

Received March 23, 1978