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Perturbation Theory for Cosine Families on Banach Spaces

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Introduction.

Let X be a Banach space and let $B(X)$ denote the set of all bounded linear operators from X into itself. A one-parameter family $\{C(t); t \in R = (-\infty, \infty)\}$ in $B(X)$ is called a *cosine family* on X if it satisfies the following conditions:

(0.1) $$C(t+s)+C(t-s)=2C(t)C(s) \quad \text{for all} \quad t, s \in R\ ;$$

(0.2) $$C(0)=I \quad \text{(the identity operator)}\ ;$$

(0.3) $$C(t)x\colon R \to X \quad \text{is continuous for every} \quad x \in X\ .$$

The associated *sine family* $\{S(t); t \in R\}$ is the one-parameter family in $B(X)$ defined by

(0.4) $$S(t)x=\int_0^t C(s)xds \quad \text{for } x \in X \text{ and } \quad t \in R\ .$$

The *infinitesimal generator* A of $\{C(t); t \in R\}$ is defined by

(0.5) $$Ax=\lim_{h\to 0} 2h^{-2}(C(h)-I)x$$

whenever the limit exists. The set of elements x for which $\lim_{h\to 0} 2h^{-2}(C(h)-I)x$ exists is the domain of A, denoted by $D(A)$.

The purpose of this paper is to prove some perturbation theorems for cosine families. That is, we give several sufficient conditions such that if A is the infinitesimal generator of a cosine family on X and B is a linear operator in X, then $\overline{A+B}$ (the closure of $A+B$) is also the infinitesimal generator of a cosine family on X. The results are related to those of Takenaka-Okazawa [4] and Travis-Webb [6] which are included in ours.

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