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The Nonvanishing Property of a Weakly Stationary Process

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Introduction.

A measurable function $f(t)$ over $-\infty<t<\infty$ with the following property is said to have the nonvanishing property: if there is a function $g(z)$, $z=t+iy$, analytic on a rectangle $a\leqq t\leqq b$, $0<y\leqq r$ (or $a\leqq t\leqq b$, $-r\leqq y<0$) and continuous on $a\leqq t\leqq b$, $0\leqq y\leqq r$ (or $a\leqq t\leqq b$, $-r\leqq y\leqq 0$) for some $r>0$ such that $f(t)=g(t)$ almost everywhere in a subinterval interior to (a, b), then $f(t)=g(t)$ should hold almost everywhere throughout (a, b). Such $f(t)$ cannot vanish in any interval unless it is almost everywhere zero over $(-\infty, \infty)$. The situation has been exhaustively studied by Levinson [5], [6], [7], [8]. One of his basic theorems is the following

THEOREM A. *Let $f(\lambda)$ be a function of $L^1(-\infty, \infty)$ and its Fourier transform be*

$$\hat{f}(t)=(2\pi)^{-1/2}\int_{-\infty}^{\infty} e^{-it\lambda}f(\lambda)d\lambda . \tag{1.1}$$

If

$$\hat{f}(t)=O(\exp(-\theta(t))) , \qquad t\to+\infty , \tag{1.2}$$

where $\theta(t)$ is a nondecreasing function over (c, ∞) for some $c>0$ such that

$$\int_c^{\infty} \theta(t)/t^2 dt=\infty , \tag{1.3}$$

then $f(\lambda)$ has the nonvanishing property.

Let $(\Omega, \mathscr{F}, P)$ be the given probability space and let $X(t, \omega)$, $\omega\in\Omega$, $-\infty<t<\infty$ be a measurable weakly stationary process with $EX(t, \omega)=0$,

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