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More on the Schur Index and the Order and Exponent of a Finite Group

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Let G be a finite group and K a field of characteristic 0. Let χ be an absolutely irreducible character of G and let $m_K(\chi)$ denote the Schur index of χ over K. In Fein and Yamada [1], we gave a theorem which relates $m_Q(\chi)$ to the order and exponent of G, where $\boldsymbol{Q}$ is the rational field. In this paper, we will give similar results for the case $K=\boldsymbol{Q}_l$, the l-adic numbers, where l is a prime. These results are easily derived from the formula of index of an l-adic cyclotomic algebra, which was obtained by the author [4], [5].

For the rest of the paper, k is a cyclotomic extension of $\boldsymbol{Q}_l$, i.e., k is a subfield of a cyclotomic field $\boldsymbol{Q}_l(\zeta')$, where ζ' is a root of unity. For a natural number n, ζ_n denotes a primitive n-th root of unity. A *cyclotomic algebra* over k is a crossed product

$$B=(\beta, k(\zeta)/k)=\sum_{\sigma\in\mathscr{G}} k(\zeta)u_\sigma\ , \qquad (u_1=1)\ , \tag{1}$$

$$u_\sigma x=\sigma(x)u_\sigma \quad (x\in k(\zeta))\ , \quad u_\sigma u_\tau=\beta(\sigma, \tau)u_{\sigma\tau}\ , \quad (\sigma, \tau\in\mathscr{G})\ , \tag{2}$$

where ζ is a root of unity, $\mathscr{G}$ is the Galois group of $k(\zeta)$ over k, and β is a factor set whose values are roots of unity in $k(\zeta)$. Put $L=k(\zeta)$. Let $\varepsilon(L)$ denote the group of roots of unity contained in L. Let $\varepsilon'(L)$ (respectively, $\varepsilon_l(L)$) denote the subgroup of $\varepsilon(L)$ consisting of those roots of unity in L whose orders are relatively prime to l (respectively, powers of l). We have $\varepsilon(L)=\varepsilon'(L)\times\varepsilon_l(L)$. Let

$$\beta(\sigma, \tau)=\alpha(\sigma, \tau)\gamma(\sigma, \tau)\ , \quad \alpha(\sigma, \tau)\in\varepsilon'(L)\ , \quad \gamma(\sigma, \tau)\in\varepsilon_l(L)\ . \tag{3}$$

Suppose that l is an odd prime. Let $\langle\theta\rangle$ denote the inertia group and ϕ a Frobenius automorphism of the extension $k(\zeta)/k$. The order e of θ has the form $e=l^te'$, $e'\,|\,l-1$. Let f denote the residue class degree of the extension $k/\boldsymbol{Q}_l$, so $\zeta_{l^f-1}\in k$.

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