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# On $2p$-fold Transitive Permutation Groups

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## Introduction.

In Yoshizawa [6], the following two theorems were proved.

Theorem A. Let $p$ be an odd prime. Let $G$ be a permutation group on a set $\Omega=\{1, 2, \cdots, n\}$ which satisfies the following condition. For any $2p$ points $\alpha_1, \cdots, \alpha_{2p}$ of $\Omega$, a Sylow $p$-subgroup $P$ of the stabilizer in $G$ of the $2p$ points $\alpha_1, \cdots, \alpha_{2p}$ is nontrivial and fixes exactly $2p+r$ points of $\Omega$, and moreover $P$ is semiregular on the set $\Omega-I(P)$ of the remaining $n-2p-r$ points, where $r$ is independent of the choice of $\alpha_1, \cdots, \alpha_{2p}$ and $0\leqq r\leqq p-2$. Then $n=3p+r$, and there exists an orbit $\Gamma$ of $G$ such that $|\Gamma|\geqq 3p$ and $G^{\Gamma}\geqq A^{\Gamma}$.

Theorem B. Let $p$ be an odd prime $\geqq 11$. Let $G$ be a permutation group on a set $\Omega=\{1, 2, \cdots, n\}$ which satisfies the following condition. For any $2p$ points $\alpha_1, \cdots, \alpha_{2p}$ of $\Omega$, a Sylow $p$-subgroup $P$ of the stabilizer in $G$ of the $2p$ points $\alpha_1, \cdots, \alpha_{2p}$ is nontrivial and fixes exactly $3p-1$ points of $\Omega$, and moreover $P$ is semiregular on the set $\Omega-I(P)$ of the remaining $n-3p+1$ points. Then $n=4p-1$, and one of the following two cases holds: (1) There exists an orbit $\Gamma$ of $G$ such that $|\Gamma|\geqq 3p$ and $G^{\Gamma}\geqq A^{\Gamma}$. (2) $G$ has just two orbits $\Gamma_1$ and $\Gamma_2$ with $|\Gamma_1|\geqq p$, $|\Gamma_2|\geqq p$ and $|\Gamma_1|+|\Gamma_2|=4p-1$, and $G^{\Gamma_i}$ is $(|\Gamma_i|-p+1)$-transitive on $\Gamma_i (i=1, 2)$. Moreover, $G^{\Gamma_i}\geqq A^{\Gamma_i}$ if $|\Gamma_i|\geqq p+3$.

In [1], E. Bannai determined all $2p$-fold transitive permutation groups in which the stabilizer of $2p$ points is of order prime to $p$, where $p$ is an odd prime. By using Theorem A and Theorem B in [6], we will improve it, namely, we will prove the following result.

THEOREM 1. *Let $p$ be an odd prime $\geqq 11$, and let $q$ be an odd prime with $p<q<p+p/3$. Let $G$ be a $2p$-fold transitive permutation group on $\Omega=\{1, 2, \cdots, n\}$. If the order of $G_{1,2,\cdots,2p}$ is not divisible by $q$, then $G$ is $S_n (2p\leqq n\leqq 2p+q-1)$ or $A_n (2p+2\leqq n\leqq 2p+q-1)$.*

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