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On the Partition Problem in an Algebraic Number Field

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Let $p(n)$ be the number of the partitions of n, then the generating function of $p(n)$ is given by

$$f(x)=\prod_{n=1}^{\infty}(1-x^n)^{-1}=1+\sum_{n=1}^{\infty}p(n)x^n \qquad (|x|<1) .$$

In 1917, Hardy and Ramanujan [1] proved the asymptotic formula for $p(n)$:

$$p(n)\sim\frac{1}{4\sqrt{3}\,n}\exp\left(\pi\sqrt{\frac{2n}{3}}\right) \qquad (n\to\infty) .$$

In 1934, Wright [11] studied the partition problem of n into k-th powers of integers. In this case, the generating function is

$$f_k(x)=\prod_{n=1}^{\infty}(1-x^{n^k})^{-1}=1+\sum_{n=1}^{\infty}p_k(n)x^n \qquad (|x|<1)$$

and Wright obtained the asymptotic formula for $p_k(n)$:

$$p_k(n)\sim\frac{A_k k^{1/2}n^{-3/2+1/(k+1)}}{(2\pi)^{(k+1)/2}(k+1)^{3/2}}\exp\left(A_k n^{1/(1+k)}\right) ,$$

where

$$A_k=(k+1)\left\{\frac{1}{k}\Gamma\left(1+\frac{1}{k}\right)\zeta\left(1+\frac{1}{k}\right)\right\}^{k/(k+1)} .$$

In 1950, Rademacher [7] suggested the problem of generalizing the partition function to algebraic number field. Three years later, in 1953, Meinardus [4] succeeded in obtaining the asymptotic formula for the partition function in a real quadratic field: Let K be a real quadratic field and define the infinite product

$$f(\tau,\tau')=\prod_{\nu}(1-e^{-\nu\tau-\nu'\tau'})^{-1} ,$$

where the product is taken over all totally positive integers ν of K, ν'

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