TOKYO J. MATH.
VOL. 2, No. 2, 1979

An Analogue of Paley-Wiener Theorem on Rank 1 Semisimple Lie Groups I

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(Communicated by T. Saito)

In the previous paper [9], we have stated some results on Paley-Wiener type theorems on semisimple Lie groups without proof. In this paper we shall give detailed proofs of those theorems.

§ 1. Notation and preliminaries.

Let G be a real reductive Lie group with compact center. We assume that G is in class $\mathscr{H}$ (cf. V. S. Varadarajan [10]). Let K be a maximal compact subgroup of G. Fix a Cartan involution θ on G induced by K. Let P be a parabolic subgroup of G, and $P=MAN$ be the associated Langlands decomposition of P. Then M is a reductive group and is in class $\mathscr{H}$, A is a vector group, which we call the split component of P, and N is the unipotent radical of P. Moreover if P is cuspidal, i.e., $\mathrm{rank}(M)=\mathrm{rank}(K_M)$ ($K_M=K\cap M$), then there exists a compact Cartan subgroup T of M and $H=TA$ is a Cartan subgroup of G. Now we denote Lie algebras by small German letters and for any real vector space V, we denote by V_c the complex vector space of V and by V^* the dual space of V. Then $\mathfrak{p}=\mathfrak{m}+\mathfrak{a}+\mathfrak{n}$ is the parabolic subalgebra of $\mathfrak{g}$ corresponding to P. In this case, $A=\exp\mathfrak{a}$, $N=\exp\mathfrak{n}$ and P is the normalizer of $\mathfrak{p}$ in G. Let $\mathscr{F}$ be the dual space of $\mathfrak{a}$, i.e., $\mathscr{F}=\mathfrak{a}^*$.

Let $\tau=(\tau_1, \tau_2)$ be a unitary double representation of K on a finite dimensional Hilbert space V. Here we assume that V satisfies the conditions in Harish-Chandra [6] § 8. Then we define the V-valued Schwartz space $\mathscr{C}(G, V)$ and the subspace of τ-spherical functions $\mathscr{C}(G, \tau)$ as usual. Moreover we denote by ${}^0\mathscr{C}(G, \tau)$ the space of τ-spherical cusp forms on G. Next let τ_M be a representation of K_M on V which is the restriction of τ to K_M. Then we can also define $\mathscr{C}(M, V)$, $\mathscr{C}(M, \tau_M)$ and ${}^\circ\mathscr{C}(M, \tau_M)$ respectively.

Received May 18, 1979.