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On the Class Number of Real Quadratic Fields $Q(\sqrt{p})$ with $p\equiv 1 \pmod 4$

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Introduction

Let p be a prime number with $p\equiv 1 (\mathrm{mod}\,4)$, and h the class number of the real quadratic field $\boldsymbol{Q}(\sqrt{p})$. Let $\varepsilon=(t+u\sqrt{p})/2$ be the fundamental unit of $\boldsymbol{Q}(\sqrt{p})$ with $\varepsilon>1$. If $p\equiv 5(\mathrm{mod}\,8)$, then P. Chowla has proved (see [1])

$$(-1)^{(h-1)/2}\frac{t}{2}\equiv(-1)^m 2^{(p-1)/4} \pmod p ,$$

and

$$\frac{((p-1)/2)!}{2^{(p-1)/4}}\equiv -(-1)^m \pmod p ,$$

where

$$m=\frac{1}{2}\left\{\frac{p-1}{4}+\sum_{s<p/4}\left(\frac{s}{p}\right)\right\} ,$$

and $\left(\frac{s}{p}\right)$ is Legendre's symbol. We shall prove a generalization of these results.

§1. Notations.

Throughout this paper we shall use the following notations.

p: a prime number with $p\equiv 1 \pmod 4$
$\boldsymbol{Q}$: the rational number field
h: the class number of the real quadratic field $\boldsymbol{Q}(\sqrt{p})$
$\varepsilon=(t+u\sqrt{p})/2$: the fundamental unit of $\boldsymbol{Q}(\sqrt{p})$ with $\varepsilon>1$
$\theta=e^{2\pi i/p}$
$\zeta=e^{\pi i/p}$

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