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Euclid's Algorithm in Pure Quartic Fields

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A finite algebraic number field K is said to be euclidean if, for any integers α and $\beta(\neq 0)$ of K, there is an integer γ of K such that $|N_K(\alpha-\beta\gamma)|<|N_K\beta|$. It is well-known that there are exactly 21 quadratic euclidean fields (see E. S. Bernes and H. P. F. Swinnerton-Dyer [1]). As for cubic fields H. Davenport [4] showed that there are only a finite number of euclidean fields which are not totally real. There are several finiteness theorems like this. H. Heilbronn [2], [3], showed that, if p is a prime then the number of cyclic euclidean fields of degree p is finite. H. Davenport [5] (cf. J. W. S. Cassels [6]) also proved the finiteness of the number of totally imaginary quartic euclidean fields.

In this paper we shall prove the following

THEOREM. *There exist only a finite number of quartic euclidean fields of the form* $Q(\sqrt[4]{m})$, *where* m *is a* 4th *power-free rational integer not expressible as* $2p^2$ *with a prime* $p\equiv 3 \pmod 8$.

In proving Theorem we can restrict our consideration to some special forms of quartic fields. Indeed for the fields $Q(\sqrt[4]{-m})$, where m is a positive integer, the finiteness follows from the result of Davenport mentioned above. Further C. J. Parry [7] proved that the class number of the field $Q(\sqrt[4]{m})$ with a positive integer m is even except those of the following forms

(I) $Q(\sqrt[4]{p})$ $p\equiv 5 \pmod 8$, $Q(\sqrt[4]{4p})$ $p\equiv 5 \pmod 8$,

(II) $Q(\sqrt[4]{p})$ $p\equiv 3 \pmod 8$, $Q(\sqrt[4]{2p})$ $p\equiv 3 \pmod 8$,
$Q(\sqrt[4]{4p})$ $p\equiv 3, 7 \pmod 8$, $Q(\sqrt[4]{8p})$ $p\equiv 3 \pmod 8$,

(III) $Q(\sqrt[4]{2p^2})$ $p\equiv 3 \pmod 8$, $Q(\sqrt[4]{2})$,

where p is a rational prime. Thus our theorem is reduced to the statement that the number of euclidean fields of the form (I) or (II) is finite, since an algebraic number field of class number greater than one is not

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