TOKYO J. MATH.
VOL. 2, No. 2, 1979

On π-uniform Vector Bundles

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(Communicated by S. Yano)

In this paper, we define a notion of "π-uniform vector bundle" over a P^1-bundle $\pi: V\rightarrow W$, where V, W are algebraic varieties. First, generalizing a result of E. Sato ([5], Proposition 3), we give a necessary and sufficient condition in order that a vector bundle over a P^1-bundle is π-uniform (Lemma). By virtue of the Lemma, we give a cohomological condition in order that a vector bundle over the trivial ruled surface $P^1\times P^1$ is decomposable (Theorem 1). Also we generalize a result of S. Shatz in [6] (Corollary, p. 106) (Theorem 2).

In [2], Schwarzenberger defined the notion of 'uniform vector bundle' on a projective space P^n. Our 'π-uniform vector bundle' is an analogue of his, and is suitable for our situation of P^1-bundle $\pi: V\rightarrow W$. In his paper on uniform vector bundles [5], E. Sato developed some methods for treating such bundles. This paper is inspired by [5].

The author would like to thank Professor K. Watanabe for his valuable suggestions and encouragement.

§1. A criterion for π-uniform vector bundles.

Let k be an algebraically closed field of arbitrary characteristic and $\pi: V\rightarrow W$ a P^1-bundle, where V, W are algebraic varieties over k. By a vector bundle E on V, we mean a locally free $\mathcal{O}_V$-sheaf module of finite rank, where $\mathcal{O}_V$ is the structure sheaf of V. We use the following notation; $h^i(V, E):=\dim_k H^i(V, E)$.

DEFINITION 1. We say that a vector bundle E on V is π-uniform, if the restriction $E|_{\pi^{-1}(p)}$ of E to $\pi^{-1}(p)$ is mutually isomorphic for any point p of W.

First the following proposition is an immediate consequence of Definition 1.

Received November 22, 1978