TOKYO J. MATH.
VOL. 2, NO. 2, 1979

Asymptotic Sufficiency of Maximum Likelihood Estimator in a Truncated Location Family

Haruyoshi MITA

Waseda University

(Communicated by H. Sunouchi)

Introduction

Let $f(x)$ be a probability density function on real line which vanishes on $(-\infty, 0]$ and twice continuously differentiable in $(0, \infty)$. We consider the case that for $\alpha \geqq 2$, $f(x) \sim Ax^{\alpha-1}$ as $x \to +0$ and $f'(x) \sim Bx^{\alpha-2}$ as $x \to +0$ $(0 < A, B < \infty)$. Let $X_1, \cdots, X_n$ be an independent identically distributed random sample of size n $(n=1, 2, \cdots)$ according to a distribution P_θ with density $f(x-\theta)$, and let $\{\hat{\theta}_n\} = \{\hat{\theta}_n(X_1, \cdots, X_n)\}$ be the maximum likelihood estimator (or MLE) of θ. In this paper we prove that under some assumptions (See Section 1.), $\{\hat{\theta}_n\}$ is asymptotically sufficient statistic for $\{P_\theta : \theta \in \Theta\}$ in the sense of LeCam [5]. Our theory of asymptotic sufficiency of MLE is based on the asymptotic properties of MLE and likelihood function, which have been studied in non-regular cases by Akahira [1], Takeuchi [6], Takeuchi and Akahira [7] and Woodroofe [9]. Asymptotic sufficiency of MLE has been discussed under the regular conditions by Kaufman [4] and LeCam [5]. In Akahira [2], asymptotic sufficiency has been discussed in a non-regular case when the density function, with a location parameter, has a compact support on R^1 and positive values at the end points.

In Section 1 notations and assumptions are stated, and in Section 2 we state some known results concerning order of consistency of MLE and $\min(X_1, \cdots, X_n)$ (cf. [1], [6], [7], [9]). In Section 3 we will show that MLE is asymptotically sufficient for $\{P_\theta : \theta \in \Theta\}$ in our non-regular case.

§1. Notations and assumptions.

Let X be a sample space whose generic point is denoted by x, $\mathscr{B}$ a σ-field of subset of X and $\{P_\theta : \theta \in \Theta\}$ a set of probability measures on

Received November 15, 1978