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A Generalization of the Fourier-Borel Transformation for the Analytic Functionals with non Convex Carrier

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Introduction

Let K be a compact set of $\boldsymbol{C}$. For an analytic functional with carrier in K, $T\in\mathscr{O}'(K)$, we define the Fourier-Borel transformation by

$$\mathscr{F}(T)(u)=\langle T_z, \exp(uz)\rangle . \tag{0.1}$$

If K is convex, it is classical that the Fourier-Borel transformation establishes a linear isomorphism of the space $\mathscr{O}'(K)$ onto the space of the entire functions of exponential type in K, $\mathrm{Exp}(\boldsymbol{C}, K)$. (Polya's representation, see for example Chapter 5 of Boas [1]. For the general theory of the Fourier-Borel transformation, see Martineau [6].) If K is not convex, this theorem is false. We shall consider in this paper the case where K is an annulus with center at the origin. Let $\lambda\neq 0$ be a fixed complex number. For $T\in\mathscr{O}'(K)$, we define the transformation $\mathscr{F}_\lambda$ by

$$\mathscr{F}_\lambda(T)(u, v)=\left\langle T_z, \exp\left(\lambda\left(uz+\frac{v}{z}\right)\right)\right\rangle . \tag{0.2}$$

This simple transformation $\mathscr{F}_\lambda$ generalizes the Fourier-Borel transformation in the case of annulus and we can determine the image of $\mathscr{O}'(K)$ under the transformation $\mathscr{F}_\lambda$ (Theorem 4.2). (Kiselman [4] and Martineau [7] considered another kind of generalizations of the Fourier-Borel transformation.)

On the other hand, let S^{n-1} be the $n-1$ dimensional sphere and $\mathscr{B}(S^{n-1})=\mathscr{A}'(S^{n-1})$ the space of hyperfunctions (analytic functionals) on the sphere. For $T\in\mathscr{B}(S^{n-1})$, Hashizume, Kowata, Minemura and Okamoto [2] defined the transformation $\mathscr{P}_\lambda$ by

$$\mathscr{P}_\lambda(T)(x)=\langle T_\omega, \exp(i\lambda\langle x, \omega\rangle)\rangle . \tag{0.3}$$

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