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The Riemann-Hilbert Problem in Several Complex Variables II

Michitake KITA

Sophia University

Introduction

In the preceding paper [6], the author proved that, in a *two-dimensional* connected Stein manifold X satisfying the condition $H^2(X, \mathbf{Z})=0$, one can solve the Riemann-Hilbert problem without apparent singularities for an arbitrary divisor D and an arbitrary representation of $\pi_1(X-D, *)$ into $GL_q(\mathbf{C})$. The purpose of the present paper is to give an example of the Riemann-Hilbert problemwhich cannot be solved without apparent singularities *by the same method as in the two-dimensional case.* More precisely, let S be a 3-dimensional polydisc; then, by a result of H. Lindel [7], there exists a special divisor D of S such that we can construct a flat vector bundle V of rank q over $S-D$ satisfying the following conditions:

1) There exists an integrable holomorphic connection ∇ on $\mathcal{O}(V)$ such that $\operatorname{Ker}\nabla=\mathbf{C}(V)$ where $\mathbf{C}(V)$ is the sheaf of germs of locally constant sections of V.

2) $\mathcal{O}(V)$ is extended to a locally free analytic sheaf $\mathcal{H}$ on $S-\operatorname{Sing}(D)$ on which ∇ is the meromorphic connection with logarithmic poles along $D\cap(S-\operatorname{Sing}(D))$. The eigenvalues $\alpha_1, \cdots, \alpha_q$ of the residue of ∇ at any point of $D-\operatorname{Sing}(D)$ are rational numbers and satisfy the inequalities $0\leqq\alpha_i<1$ for $i=1, \cdots, q$.

3) $\mathcal{H}$ is extended *uniquely* to a coherent analytic sheaf $\tilde{\mathcal{H}}$ on S satisfying $\tilde{\mathcal{H}}^{[1]}=\tilde{\mathcal{H}}$, but $\mathcal{H}$ *cannot be extended to any locally free analytic sheaf on* S, where $\tilde{\mathcal{H}}^{[1]}$ is the *first absolute gap-sheaf* of $\tilde{\mathcal{H}}$ (for the definition of absolute gap-sheaves, see [9]).

It seems to the author that if, in three dimension, one wants to solve the Riemann-Hilbert problem without apparent singularities even in the local sense, one should study in detail the Manin extension (See 1.2.) and the structure of vector bundles which are meromorphic along a divisor (see [3]), and should take deeper consideration on the equation

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