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A Verification for Non-existence of Movable Branch Points of Six Painlevé Transcendents by Formula Manipulations

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§1. Statement of the problem.

Suppose that the differential equation

$$w''=F(z, w, w') \qquad ('=d/dz)$$

satisfies that F is rational in w', algebraic in w and analytic in z. Among such class of equations, it is well-known that any irreducible equation without movable critical points (i.e., the branch points and essential singular points) must be one of the following six equations, which are called Painlevé equations:

$$w''=6w^2+z\,, \tag{1}$$

$$w''=2w^3+zw+\alpha\,, \tag{2}$$

$$w''=\frac{1}{w}(w')^2-\frac{1}{z}w'+\frac{1}{z}(\alpha w^2+\beta)+\gamma w^3+\frac{\delta}{w}\,, \tag{3}$$

$$w''=\frac{1}{2w}(w')^2+\frac{3}{2}w^3+4zw^2+2(z^2-\alpha)w+\frac{\beta}{w}\,, \tag{4}$$

$$w''=\left(\frac{1}{2w}+\frac{1}{w-1}\right)(w')^2-\frac{1}{z}w'+\frac{(w-1)^2}{z^2}\left(\alpha w+\frac{\beta}{w}\right)+\frac{\gamma w}{z}+\frac{\delta w(w+1)}{w-1}\,, \tag{5}$$

$$w''=\frac{1}{2}\left(\frac{1}{w}+\frac{1}{w-1}+\frac{1}{w-z}\right)(w')^2-\left(\frac{1}{z}+\frac{1}{z-1}+\frac{1}{w-z}\right)w' \\ +\frac{w(w-1)(w-z)}{z^2(z-1)^2}\left\{\alpha+\frac{\beta z}{w^2}+\frac{\gamma(z-1)}{(w-1)^2}+\frac{\delta z(z-1)}{(w-z)^2}\right\}\,. \tag{6}$$

Conversely, the general solutions of these equations, called Painlevé

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