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Solutions of $x''=t^{\alpha\lambda-2}x^{1+\alpha}$ with Movable Singularity

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Introduction

As in the previous paper [1], we consider here a second order non-linear differential equation

$$(1) \qquad x''=t^{\alpha\lambda-2}x^{1+\alpha}\ , \qquad '=d/dt\ , \qquad \alpha>0\ , \qquad \alpha\lambda>1\ ,$$

in a domain

$$G: \qquad 0<t<\infty\ , \qquad 0\leqq x<\infty\ .$$

As we restrict ourselves entirely within the real domain, any real power of a nonnegative-valued variable should be regarded as representing its nonnegative-valued branch. So, for example,

$$t^{\alpha\lambda-2}>0\ , \qquad x^{1+\alpha}\geqq 0$$

in G.

The solutions of (1) to be considered here are those which satisfy the "initial condition"

$$\lim_{t\to 0} x=a\ , \qquad \lim_{t\to 0} x'=b\ , \qquad 0<a<\infty\ , \qquad |b|<\infty\ .$$

Such solutions will be denoted by $\phi(t, a, b)$. The object of this paper is to show that each $\phi(t, a, b)$ has, in general, a movable singularity and to obtain the explicit expression of $\phi(t, a, b)$ valid in the vicinity of its movable singularity.

To do this, we have to make use of some of the results obtained in [1]. This section is devoted to the brief description of them.

The equation (1) has a solution

$$x=\psi(t)=[\lambda(\lambda+1)]^{1/\alpha}t^{-\lambda}\ .$$

For any solution $x(t)$ of (1), let us define a function $y(t)$ by

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