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On Unimodal Linear Transformations and Chaos II

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Introduction

In part II we consider the general unimodal linear transformations, that is, a family of maps from [0, 1] into itself which take the extremum at c for some $c\in(0, 1)$ and are linear on each intervals $[0, c]$ and $[c, 1]$. It is not difficult to show that, except for some trivial exceptions, the consideration of the general unimodal linear transformations defined above can be reduced to that of the special class $\{f_{a,b};\, b>1,\, ab>1,\, a+b\geqq ab\}$ defined in the following way:

$$f_{a,b}(x)=\begin{cases} ax+\dfrac{a+b-ab}{b} & \text{for } \ 0\leqq x\leqq 1-\dfrac{1}{b} \\ -b(x-1) & \text{for } \ 1-\dfrac{1}{b}\leqq x\leqq 1\,. \end{cases}$$

In the cases which will be discussed below there will appear phenomena called "window" and "islands", which did not occur in the case $a=b$ of part I. Let us explain these cases, dividing the case $b=4$ into several classes according to the behavior of the corresponding $f_{a,b}$.

1) The case of $0<a<1/4$ (that is, the case of $ab<1$).

In this case, there exists a unique periodic orbit with period 2 and all points except the fixed point approach this periodic orbit. So this class is a stable class, and we omit this class from further consideration.

2) The case of $a=1/4$ (that is, the case of $ab=1$).

Let $A_0=[0, 3/4]$ and $A_1=[13/16, 1]$, then we have $f_{a,b}A_0=A_1$, $f_{a,b}A_1=A_0$, and $f^4_{a,b}|_{A_i}$ is the identity map on $A_i (i=0, 1)$ and every orbit starting from $(3/4, 13/16)-\{4/5\}$ enters into $A_0\cup A_1$. So, this class is also stable.

3) The case of $1/4<a\leqq 4/15$ (that is, the case of $ab>1$, $(a+b-ab)/b\geqq b/(b+1)$).

There exist a natural number m and intervals $A_0, A_1, \cdots, A_{2^m-1}$ such that $f_{a,b}A_i=A_{i+1}$ for $0\leqq i\leqq 2^m-2$ and $f_{a,b}A_{2^m-1}=A_0$, and every orbit starting from $[0, 1]-\bigcup_{i=0}^{2m-1} A_i$ (except the fixed point of $f^{2m}_{a,b}$) enters into

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