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On Unimodal Linear Transformations and Chaos I

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Introduction

Recently, much work has been done to investigate for a parametrized family of continuous mappings on an interval how the structure of orbits for such maps changes as the parameters vary. These investigations are motivated by the desire to analyze chaotic phenomena frequently observed in nature, in studying how the behavior of dynamical systems (appearing in various models such as Lorentz models, ecological models or models describing chemical reactions and so on) is influenced by the change in characteristic parameters and turns into a turbulent state.

For the case of one-dimensional systems, attempts have been made to characterize how the change in parameter values gives rise to the existence of unstable non-periodic orbits; such characterization is done by describing the nature of periodic points that appear; for instance, by the appearance of periodic points of period 3 [5]. If the mapping in question possesses an invariant measure, the existence of unstable non-periodic orbits suggests that the mapping may have ergodic or mixing properties. Furthermore, it is possible to measure the "size" of the set of points having non-periodic orbits, in terms of the "size" of the support of the invariant measure. It seems to be also natural to explain the appearance of the so-called "window" phenomena, observing that in such cases the invariant measure is not absolutely continuous with respect to the Lebesgue measure on the interval.

In this paper, we concentrate ourselves on the study of unimodal linear transformations on the unit interval [0, 1], which are the simplest one-dimensional models. Let us define a family $\{f_\mu\}$ of mappings of the unit interval [0, 1] in the following way, where the parameter μ is determined by a pair of real numbers (a, b):

$f_\mu(x)$ is linear with the slope a on the sub-interval $[0, c]$ for some $c (0<c<1)$, linear with slope $-b$ on the sub-interval $[c, 1]$, and is

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