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The Microlocal Structure of Weighted Homogeneous Polynomials Associated with Coxeter Systems I

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Introduction

Let E be an l-dimensional Euclidean space with an orthonormal basis $\{e_i\}$ and E^* its dual with the dual basis $\{\xi_i\}$. Let, further, W be a finite group of GL(E) generated by reflections. Such a group is completely classified and forms a Coxeter system (W, S) for an appropriate set S of generators [1]. Let R be the subalgebra of the symmetric algebra $S(E^*)$ whose elements are invariant under the action of W. As is known, there exist algebraically independent homogeneous elements $x_1, \cdots, x_l$ of R such that $R=\boldsymbol{R}[x_1, \cdots, x_l]$. Let $D(\xi)$ be the product of linear functions defining the hyperplanes of reflections of W. Then $D(\xi)^2$ is represented as a polynomial of $x_1, \cdots, x_l$. We denote it by $f_W(x)$ and call it the generalized discriminant in this paper.

Let us consider the space $X=(E^*/W)^c$, the complexification of the quotient space of E^* by W, whose coordinate ring is $\boldsymbol{C}\otimes R$. Then

$$m_{ij}(x)=\frac{1}{2}\sum_{k=1}^{l}\frac{\partial x_i}{\partial\xi_k}\frac{\partial x_j}{\partial\xi_k}\qquad(1\leqq i,\ j\leqq l)$$

belong to R and the vector fields

$$X_i=\sum_{j=1}^{l}m_{ij}(x)\frac{\partial}{\partial x_j}\qquad(1\leqq i\leqq l)$$

leave $f(x)=f_W(x)$ invariant. More precisely, we have

$$X_i f(x)=c_i(x)f(x)$$

with certain polynomials $c_i(x)\in R$. Furthermore, $X_1, \cdots, X_l$ form a free basis of the Lie algebra of vector fields leaving the set $\{x; f(x)=0\}$ invariant ([7]).

In this paper, we shall study the microlocal structure of the $\mathscr{D}_X$-Module

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