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A Numerical Investigation on Cumulative Sum of the Liouville Function

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Let $\lambda(n)$ be the Liouville function defined as $\lambda(n)=(-1)^{\nu}$, where ν is the number of prime factors of a positive integer n, multiple factors being counted according to their multiplicity. Thus $\lambda(1)=1$, $\lambda(2)=-1$, $\lambda(3)=-1$, $\lambda(4)=1$, $\lambda(5)=-1$, $\lambda(6)=1$, $\lambda(7)=-1$, $\lambda(8)=-1$, $\lambda(9)=1$, $\lambda(10)=1, \cdots$.

We put

$$L(x)=\sum_{n=1}^{x}\lambda(n) .$$

In this paper we assume x to be a positive integer. Thus $L(1)=1$, $L(2)=0$, $L(3)=-1$, $L(4)=0$, $L(5)=-1$, $L(6)=0$, $L(7)=-1$, $L(8)=-2$, $L(9)=-1$, $L(10)=0, \cdots$.

The object of this note is to report some numerical results obtained by the author on $L(x)$, $x\leqq 10^9$, especially on how $L(x)$ changes its sign as x increases from 1 to 10^9.

For convenience we divide the integers $1-10^9$ into subregions each consisting of 10000 consecutive integers.

1−10000. In this region, $L(x)=0$ only for $x=2, 4, 6, 10, 16, 26, 40, 96, 586$; $L(x)>0$ only for $x=1$.

10001−906150000. Always $L(x)<0$.

906150001−906160000. $L(x)=0$ for 54 values of x, the first of which is 906150256; $L(x)>0$ for 1529 values of x, the first of which is 906150257.

906160001−906180000. Always $L(x)<0$

906180001−906190000. $L(x)=0$ for 16 values of x; $L(x)>0$ for 9612 values of x.

906190001−906200000. $L(x)=0$ for 75 values of x; $L(x)>0$ for 7784 values of x.

906200001−906210000. $L(x)=0$ for 22 values of x; $L(x)>0$ for 9643 values of x.

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