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On closed subalgebras between A and H^∞

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Introduction

Let D, $\bar{D}$, H^∞ and A be respectively the open unit disc in the complex plane C, the closed unit disc, the algebra of all bounded analytic functions on D and the algebra of all continuous functions on $\bar{D}$ which are analytic in D. The norm of a function f in H^∞ or A is the supremum of $|f|$ on D. Let $L^\infty(T)$ be the algebra of all essentially bounded, Lebesgue measurable functions on the unit circle T in C. The norm of a function f in $L^\infty(T)$ is the essential supremum of $|f|$ on T. We can regard H^∞ as a closed subalgebra of $L^\infty(T)$ by considering the radial limit of a function of H^∞, due to the Fatou's theorem. A convenient reference book for the basic facts about these algebras in Hoffman [2].

It is well known that the Douglas' conjecture about the closed subalgebras between H^∞ and $L^\infty(T)$ was solved affirmatively by Chang [1] and Marshall [2] showing that every closed subalgebra of $L^\infty(T)$ containing H^∞ is unequely determined by its maximal ideal space. However, when we have an intention to characterize the closed subalgebras of H^∞ containing A, the role of the maximal ideal space is no longer so definitive as in the case of the Douglas algebras in $L^\infty(T)$, because of the existence of the two closed subalgebras with the same maximal ideal space.

In this paper we shall construct these algebras, which seem to be never known before. The method of this construction is essentially due to Scheinberg [4]. In [5] we shall show further results, i.e., (i) these algebras have the same Silov-boundary which is coincident with the Silov-boundary of H^∞. (ii) These algebras are not log-modular in $L^\infty(T)$. (iii) Each unit-ball of these algebras is the closed convex hull of its Blaschke products.

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