

An Analogue of the Theorem of Paley-Wiener Type on the Universal Covering Group of de Sitter Group

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Introduction

The purpose of this paper is to prove the theorem of Paley-Wiener type on the universal covering group of de Sitter group (Theorem 3.4). Theorems of this type on semisimple Lie groups have been proved in several cases: L. Ehrenpreis and F. I. Mautner [5] is the first that proved theorems of Paley-Wiener type for $SL(2, \mathbf{R})$. For a general non-compact semisimple Lie group, this is not proved but there are some papers on theorems of similar type; [17] for compact groups, [7], [11] for symmetric spaces and [1], [14], [16], for certain another function spaces. And recently some characterizations of the Fourier image of C^∞ -function with compact support on a rank 1 semisimple Lie group were given in [2], [6], [12]. But in the case when G is the universal covering group of de Sitter group we can give a more explicit characterization than that of [2], [6], [12].

The techniques used in the proof of Theorem 3.4 are similar to that of [11]. But, in our case, functions which appear in the proof have more singularities. Most difficulties of the proof are the arguments on these singularities. Theorem 3.4 of [13] is the key result. By using this theorem, we can reduce the proof to the case where above singularities are absent.

We divide this paper into three parts. In the first section, we give the realization of representations of G and establish certain elementary properties of the matrix coefficients of these representations. We need these properties to describe and to prove Theorem 3.4. In the second section, we give the definition of the Fourier transform and Plancherel formula. In section three, we state the main theorem (Theorem 3.4) and prove this.

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