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Analytic Functionals on the Lie Sphere

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Introduction

Suppose $S^1=\{z\in C; |z|=1\}$ is the unit circle. Let us denote by $L^2(S^1)$ the Hilbert space of square integrable functions on S^1 equipped with the inner product $(f, g)_{L^2(S^1)}=(f, \bar{g})_{S^1}$, where $(\ ,\ )_{S^1}$ is the bilinear form defined as follows:

$$(0.1)\qquad (f, g)_{S^1}=\frac{1}{2\pi}\int_0^{2\pi} f(e^{i\theta})g(e^{i\theta})d\theta\ .$$

Let us denote by $\mathscr{H}^{(m)}(S^1)$ the one dimensional subspace of $L^2(S^1)$ spanned by the exponential function $e^{im\theta}$. Then we have the direct sum decomposition:

$$(0.2)\qquad L^2(S^1)=\bigoplus_{m\in Z}\mathscr{H}^{(m)}(S^1)$$

and the orthogonal projection of $L^2(S^1)$ onto $\mathscr{H}^{(m)}(S^1)$ is given by

$$(0.3)\qquad f(e^{i\theta})\longmapsto c_m e^{im\theta}\ ,$$

where

$$(0.4)\qquad c_m=\frac{1}{2\pi}\int_0^{2\pi} f(e^{i\theta})e^{-im\theta}d\theta$$

is the m-th Fourier coefficient of f.

More generally, suppose S^{n-1} is the $n-1$ dimensional unit sphere. $d\Omega_n$ denotes the invariant measure on S^{n-1} and Ω_n is the volume of S^{n-1}. Denote by $L^2(S^{n-1})$ the Hilbert space of square integrable functions on S^{n-1} equipped with the inner product $(f, g)_{L^2(S^{n-1})}=(f, \bar{g})_{S^{n-1}}$, where $(\ ,\ )_{S^{n-1}}$ is the bilinear form defined as follows:

$$(0.5)\qquad (f, g)_{S^{n-1}}=\frac{1}{\Omega_n}\int_{S^{n-1}} f(\omega)g(\omega)d\Omega_n(\omega)\ .$$

If we denote by $\mathscr{H}^k(S^{n-1})$ the space of spherical harmonics of degree k,

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