TOKYO J. MATH.
VOL. 4, No. 1, 1981

Average Version of Artin's Conjecture in an Algebraic Number Field

Shigeki EGAMI

Keio University

(Communicated by T. Saito)

Artin's conjecture on primitive root was proved by Hooley [1] under the generalized Riemann hypothesis for certain family of Dedekind Zeta functions. Its generalization to arbitrary number fields was also obtained by Weinberger [2] and Lenstra [3]. In another direction, Goldfeld [4] obtained a "large sieve type" result for the rational case without Riemann hypothesis. In this paper we shall show that his method can be applied to obtain the similar result for the case treated in [2], [3].

Let K be an algebraic number field of degree n, and q be a fixed integral divisor which contains all the infinite primes of K. For $A>0$ we define a set of non-zero integers of K by

$$B'(A)=\{\alpha;\ |\alpha^{(\nu)}|\leqq A,\ (\nu=1,\ \cdots,\ n)\}\ ,$$

where $\alpha^{(\nu)}$ denotes a conjugate element of α. We write $\alpha\sim\beta$ for integers α, β of K if both α and β generate the same principal ideal. We denote by $B(A)$ a fixed collection of the representatives taken from each equivalence class of $B'(A)/\sim$. Let h be a ray class modulo q, and define

$$(1)\qquad N_{\alpha,q,h}(x)=N_\alpha(x) =\mathrm{card}\,\{\mathfrak{p};\ N\mathfrak{p}\leqq x,\ \mathfrak{p}\in h,\ \alpha \text{ is a primitive root modulo } \mathfrak{p}.\}\ ,$$

and

$$g(K)=\sup_{1\leqq x<\infty}\sum_{N\mathfrak{a}\leqq x}1/x$$

where $\mathfrak{a}$ denotes an integral ideal of K. We denote

$$(2)\qquad \mathrm{Li}\,(x)=\int_2^x\frac{dt}{\log t}\ .$$

Received August 29, 1980