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Formulae for the Values of Zeta and L-functions at Half Integers

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Introduction

Ramanujan's formula for the Riemann zeta function at half integers was formulated and established by Y. Matsuoka [3]. And he deduced interesting representations for $\zeta(1/2)\zeta(2a-1/2)$ and $\zeta(-1/2)\zeta(2a+1/2)$, where $\zeta(s)$ is the Riemann zeta function and a is an integer greater than 1. Recently, the author [4] generalized the results of Y. Matsuoka [3]. The purpose of this paper is to derive similar results for the values of zeta and L-functions.

§ 1. Notations and results.

Throughout this paper, we assume that a is a positive integer and b is a non-negative integer. As usual, N and Q denote the set of natural numbers and the field of rational numbers, respectively. Let χ be a primitive non-principal character modulo k and $L(s, \chi)$ the Dirichlet L-function associated with χ. Denote by $\bar{\chi}$ the character conjugate to χ. Let $B_{n,\chi}$ denote the n-th Bernoulli number corresponding to χ in the sense of Leopoldt. For any positive integer n, we set

$$g_1(a, b, \chi; n)=\sum_{\substack{n_1n_2n_3|n \\ (n_1,n_2,n_3)\in N^3}} \chi(n_1)\bar{\chi}(n_2)n_1^{-b-1/2}n_2^{2a-1}n_3^{2a-b-3/2}$$

and

$$g_2(a, b, \chi; n)=\sum_{\substack{n_1n_2n_3|n \\ (n_1,n_2,n_3)\in N^3}} \chi(n_1)\bar{\chi}(n_2)n_1^{-b-1/2}n_2^{2a}n_3^{2a-b-1/2} .$$

For brevity we will often write

$$g_i(n)=g_i(a, b, \chi; n) \quad (i=1, 2) .$$

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