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Dynamical System with Continuous States and Relative Free Energy

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Introduction

In this article, we are concerned with a dynamical system on the 1-dimensional integer lattice the state space of which is compact (not necessarily discrete). When the state space of the system is not discrete the usual free energy may diverge and hence instead of it we consider the relative free energy which we introduced in [3]. In § 2, we discuss the variational principle in this system with respect to relative free energy and we show that the measure which minimizes relative free energy is obtained as a unique solution of an eigenvalue problem of a transfer operator, if the potential has not so long range (Theorem 2.1).

Moreover we discuss its cluster property and we prove that the equilibrium measure is mixing under the same assumption on the potential as the above (Theorem 3.1), and especially it is weak Bernoulli, if the potential has finite range (Theorem 3.2).

In the former paper [3], we discussed time evolution of a Markov process μ_t of a speed change model on our dynamical system and we proved that the relative free energy of μ_t decreases according to time evolution. Combining these results with Theorem 2.1 in this article, it follows that every initial state converges to the equilibrium state, if the Gibbs measure is unique.

§ 1. Construction of a shift invariant measure.

Let Ω_0 be a compact Hausdorff space with the second countability axiom and let $\mathscr{B}_0$ be its topological Borel field. We suppose that a probability measure ν_0 and a metric d_0 with $d_0(x, y)\leqq 1$, $x, y\in\Omega_0$ are endowed with $(\Omega_0, \mathscr{B}_0)$. Let $(\tilde{\Omega}, \tilde{\mathscr{B}}, \tilde{\nu})$ be the 1-sided countable product

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