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Characterizations of the Ranges of Wave Operators for Symmetric Systems and an Application

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§1. Introduction.

The spectral and scattering theory for partial differential operators has been investigated extensively (see [1], [8], [10], [11], [13], [14], and references there). But certain ellipticity has been assumed to ensure the completeness of wave operators. Such an assumption excludes, for example, the equation of magnetgasdynamics which has a characteristic root like ξ_1 (see [3, p. 298]). The aim in this paper is to remove such an ellipticity assumption.

In [14] the author established the existence and completeness of the wave operators for "symmetric systems", which are rather general ones including symmetric hyperbolic systems and Schrödinger equations. The existence of the wave operators was established under the condition that perturbations are short range; no additional condition was assumed. In establishing the completeness, however, certain ellipticity was further assumed in order to apply the compactness argument. Without such an ellipticity assumption we shall show in this paper that the wave operators are complete in a weak sense. It is shown, for example, that the ranges of the wave operators for symmetric hyperbolic systems with characteristic roots of constant multiplicity are scattering subspaces. (For an abstract scattering theory which is not based on the subspace of absolute continuity, see [18].) In order to establish it we apply the method recently invented by Enss [5], which does not essentially rely on the compactness argument and is more direct than the method of Kato-Kuroda. The crucial tool which makes this application possible is the L_2-boundedness theorem of Calderon-Vaillancourt [4].

Now, we explain notations in order to state the results. We write $D_j=-i\partial/\partial x_j$, $x=(x_1, \cdots, x_n)\in \boldsymbol{R}^n$, $\langle x\rangle=(1+|x|^2)^{1/2}$, $\langle D\rangle^2=1+D_1^2+\cdots+D_n^2$, $D^\alpha=D_1^{\alpha_1}\cdots D_n^{\alpha_n}$, where $\alpha=(\alpha_1, \cdots, \alpha_n)$ is a multi-index. For G_1 and G_2

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