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Spectral Property of Goursat Problems

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Introduction

In the study of Goursat problems, a "spectral radius" plays an important role (see [1]). For example, let us take the Goursat problem studied by J. Leray in [4]

$$(1)\qquad \begin{aligned} &\varepsilon u_{x_1x_2}=u_{x_1x_1}+u_{x_2x_2}+h(x_1, x_2)\\ &u(x_1, 0)\equiv u(0, x_2)\equiv 0 \end{aligned}$$

where $(x_1, x_2)\in C^2$ and $h(x_1, x_2)$ is analytic in a neighborhood of the origin. The spectral radius of (1) is 2 and if the condition $|\varepsilon|>2$ is satisfied (1) has one and only one analytic solution in a neighborhood of the origin.

On the other hand the result when $|\varepsilon|<2$ is rare. As far as the author knows, the best work known hitherto is that of Leray's in [4]. He introduced the function

$$\rho(t)=\liminf_{k\to\infty}\ |\sin(k\pi t)|^{1/k}$$

with $\varepsilon=2\cos(\pi t)$ and showed that if the condition $\rho(t)>0$ is satisfied, (1) has one and only one analytic solution.

In this paper we shall extend Leray's results and give sufficient conditions for the existence and uniqueness of the solution when $\rho(t)=0$. We also give the eigenfunction expansion of the solution and using this we shall study the regularity of the solution with respect to ε.

§1. Notations and results.

Let $x=(x_1, \cdots, x_d)$ be the variable in complex d-dimensional space C^d with $d\geqq 2$. We use multi-indices $\alpha=(\alpha_1, \cdots, \alpha_d)\in Z^d$ with $|\alpha|=\alpha_1+\cdots+\alpha_d$ where Z denotes the set of integers. We denote by

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