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Monodromy Group of Appell's System (F_4)

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Introduction

Appell's hypergeometric function $F_4(\alpha, \beta, \gamma, \gamma'; x, y)$ is defined by

$$F_4(\alpha, \beta, \gamma, \gamma', x, y)=\sum_{m,n=0}^{\infty}\frac{(\alpha, m+n)(\beta, m+n)}{(\gamma, m)(\gamma', n)(1, m)(1, n)}x^m y^n$$

where (a, k) denotes the factorial function;

$$(a, k)=\Gamma(a+k)/\Gamma(a) .$$

We assume $\gamma, \gamma'\neq 0, -1, -2, \cdots$ throughout this paper. This power series converges in the domain $\{(x, y)\in C^2; \sqrt{|x|}+\sqrt{|y|}<1\}$ and satisfies the following system of partial differential equations.

$$(F_4)\quad \begin{aligned} &x(1-x)r-y^2t-2xys+[\gamma-(\alpha+\beta+1)x]p-(\alpha+\beta+1)yq-\alpha\beta z=0 \\ &y(1-y)t-x^2r-2xys+[\gamma'-(a+\beta+1)y]q-(\alpha+\beta+1)xp-\alpha\beta z=0 . \end{aligned}$$

Where z is the unknown function and

$$p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}, r=\frac{\partial^2 z}{\partial x^2}, s=\frac{\partial^2 z}{\partial x\partial y}, t=\frac{\partial^2 z}{\partial y^2} .$$

This system (F_4) is equivalent to a completely integrable system of linear differential equations of rank 4 whose coefficients are rational functions with poles only on $L=L_1\cup L_2\cup L_3\cup C$ and A in P^2, where

$$L_1=\{x=0\} , \quad L_2=\{y=0\} , \quad L_3=\text{line at infinity}$$
$$C=\{(x-y)^2-2(x+y)+1=0\} , \quad A=\{x+y=1\} .$$

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