

The Microlocal Structure of Weighted Homogeneous Polynomials Associated with Coxeter Systems, II

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Introduction

This paper is a continuation of the previous paper [9]. We retain the notation used there.

Let (W, S) be a finite Coxeter system. If the rank of W is l , there exist l -number of algebraically independent W -invariant polynomials x_1, \dots, x_l which freely generate the W -invariant ring. Let $f_W(x_1, \dots, x_l)$ be the square of an anti-invariant of W . Then there exist vector fields X_1, \dots, X_l such that they form a free basis of the Lie algebra of vector fields thereby the set $\{x \in C^l; f_W(x) = 0\}$ being left invariant. In particular, we have $X_i f_W = c_i(x) f_W$ with a certain polynomial $c_i(x)$ ($i=1, \dots, l$). We studied in [9] the microlocal structure of the \mathcal{D}_{C^l} -Module

$$\mathcal{N}'_\alpha = \mathcal{D}_{C^l} / \sum_{i=1}^l \mathcal{D}_{C^l} (X_i - \alpha c_i(x)) \quad (\alpha \in C).$$

The main purpose of this paper is to determine the holonomy diagram of the system \mathcal{N}'_α for an irreducible finite Coxeter system (except of types E_7 and E_8), which gives enough information for the system \mathcal{N}'_α .

In connection with the study, we shall also obtain rather computational results concerning the basic invariants x_1, \dots, x_l , which are complementary to our study but may be useful to the theory of logarithmic poles developed by Professor K. Saito (cf. [4]). Since we have only intermediate results when W is of type E_7 or E_8 , we treat in this paper the Coxeter systems of types $A_l, B_l, D_l, E_6, F_4, G_2, H_3, H_4, I_2(p)$.

§ 1. The holonomy diagram.

(1.1) We begin with explaining the holonomy diagram of a holonomic system. The holonomy diagram gives enough information for the holonomic system. A typical application will be seen in [3].