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Metrical Theory for a Class of Continued Fraction Transformations and Their Natural Extensions

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Introduction

In this article we consider the class of continued fraction transformations $\{f_\alpha\}$ including the transformations associated with continued fractions to the nearest integer, singular continued fractions and with simple continued fractions. Here f_α, $1/2\leqq\alpha\leqq 1$, is defined by

$$f_\alpha(x)=\left|\frac{1}{x}\right|-\left[\left|\frac{1}{x}\right|+1-\alpha\right] \qquad \text{for} \quad x\neq 0,\ x\in[\alpha-1,\alpha)\ .$$

Many results concerning the metrical theory for the simple continued fractions had been given by Gauss, Lévy, Khintchine, etc., (see Billingsley [1]). On the other hand, the metrical theory of continued fractions to the nearest integer or of singular continued fractions has been discussed by Rieger [7], [8] and [9], in which he obtained among other things the invariant measures for these transformations.

In contrast with $\{f_\alpha\}$, recently Ito and Tanaka [3] considered the class of transformations $\{S_\alpha\}$ including those associated with the restriction to the real axis of Hurwitz' complex continued fractions and of simple continued fractions. Here S_α, $1/2\leqq\alpha\leqq 1$, is defined by

$$S_\alpha(x)=\frac{1}{x}-\left[\frac{1}{x}+1-\alpha\right] \qquad \text{for} \quad x\neq 0, x\in[\alpha-1,\alpha)\ ;$$

they have obtained the absolutely continuous invariant measures and computed entropies $h(S_\alpha)$ with respect to them for the cases of $1/2\leqq\alpha\leqq(\sqrt{5}-1)/2$.

In this note, first we will show the convergence of expansions with respect to f_α and some fundamental properties. The essential property of $\{f_\alpha\}$ is that the denominators q_n of the n-th approximants with respect to f_α are always positive in contrast with the case of S_α. Next we will

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