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Invariant Measures for Homeomorphisms with Weak Specification

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Introduction

In this paper one considers the space of measures provided with the weak topology. In [7, 8], K. Sigmund discussed some categories in the space of invariant measures for homeomorphisms satisfying specification. The ingredient of his proofs is in the densely periodic property of homeomorphisms with specification. It is known that weak specification for homeomorphisms is strictly weaker than specification.

Our aim is to show that the results of K. Sigmund hold for homeomorphisms satisfying weak specification (Theorems 1 and 3). The idea of proofs is in constructing the property "smallest sets" (See § 2.) that is found in the weak specification property.

§ 1. Definitions and main results.

Let X be a compact metric space with metric d and $\mathfrak{M}(X)$ be the space of Borel probability measures of X with metric $\bar{d}$ which is compatible with the weak topology, where $\bar{d}$ is defined by

$$\bar{d}(\mu, \nu)=\inf\{\varepsilon;\ \mu(B)\leqq\nu(\{x\in X;\ d(x, B)\leqq\varepsilon\})+\varepsilon \quad \text{and} \quad \nu(B)\leqq\mu(\{x\in X;\ d(x, B)\leqq\varepsilon\})+\varepsilon \ \text{for all Borel sets } B\}$$

(p. 9 of [5] or p. 238 of [3]).

Define a point measure $\delta(x)$ by $\delta(x)(B)=1$ if $x\in B$ and $\delta(x)(B)=0$ if $x\notin B$ (Borel sets B), and denote by $B(x, \varepsilon)$ an ε-closed ball about x in X. For arbitrary finite points $x_i\in X$ and $\mu_i\in\mathfrak{M}(X)$ $(1\leqq i\leqq n)$ with $\mathrm{card}\{1\leqq i\leqq n;\ \mu_i(B(x_i, \varepsilon))<1\}/n<\varepsilon$, we get easily $\bar{d}(1/n\sum_{i=1}^{n}\delta(x_i), 1/n\sum_{i=1}^{n}\mu_i)<\varepsilon$. It is clear that the map $x\rightarrow\delta(x)$ $(x\in X)$ is a homeomorphism from X onto a subset of $\mathfrak{M}(X)$.

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