

Note on Differentiable Maps and Liapunov Index

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ABSTRACT. We prove that if a differentiable map f has a negative Liapunov index at a point x , then there exists an attractive neighborhood of x , and prove in addition that, if f is a unimodal map on an interval which has negative Schwarzian derivative, and satisfies the same condition as above, then there exists an attractive periodic point which absorbs x .

Introduction

Ruelle [1] and Pesin [2] investigated the properties of dynamical systems by means of Liapunov index. The most remarkable result among these is Pesin's work, in which he constructed a Markov partition for the dynamical systems with smooth invariant measure having non-zero Liapunov index. On the other hand, without assuming the existence of smooth invariant measure, Ruelle succeeded in the construction of the stable and the unstable manifold. In our paper, we will construct attractive domains for differentiable maps. Unfortunately, it will be difficult to calculate the Liapunov indices exactly. But we hope that the study of Liapunov indices will enable us to classify ω -limit sets of dynamical systems, and will help us to obtain characterizations of orbits moving densely in the space, characterizations of orbits getting absorbed in a strange attractor, or characterizations of attractive periodic points. This paper is intended for the first step toward the realization of this hope.

The reason why we treat differentiable maps rather than diffeomorphisms is that we want to treat the logistic equation in our analysis. Although this equation is one-dimensional, its orbits exhibit the most complicated behavior. One of the most important results on this map was given by Guckenheimer [3].