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On an Average of $\omega(n)$ with Respect to Some Sets of Composite Integers

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Throughout this paper we shall use the following notations:
$\boldsymbol{N}$: the set of all positive integers,
$\boldsymbol{P}$: the set of all rational prime numbers,
$\boldsymbol{N}(x)=\{n\in\boldsymbol{N};\ n\leqq x\}$ (for x: real),
$S(x)$: a subset of $\boldsymbol{N}(x)$,
$\#(S(x))$: the cardinal number of $S(x)$,
$\omega(n)$: the number of distinct prime factors of n,
$\Omega(n)$: the total number of prime factors of n,
$\|n\|$: $\min_{p\in\boldsymbol{P}}(|n-p|)$, i.e., the distance from n to its nearest prime.
The letters p, q will always denote prime numbers. We shall write $\log_2 x=\log\log x$ and $\log_3 x=\log\log\log x$, and use $\pi(x)$, $\pi(x;k,l)$ and $\mathrm{Li}(x)$ in the usual sense.

§1. Statement of results.

Since the value of $\omega(n)$ or that of $\{\Omega(n)-\omega(n)\}$ fluctuates irregularly, we shall observe

$$V(S(x))=\frac{\sum_{n\in S(x)}\omega(n)}{\#(S(x))}\,,\qquad W(S(x))=\frac{\sum_{n\in S(x)}\{\Omega(n)-\omega(n)\}}{\#(S(x))}\,,$$

each of which can be regarded as an average of $\omega(n)$ or that of $\{\Omega(n)-\omega(n)\}$ for a given subset $S(x)$. For $S(x)=\boldsymbol{N}(x)$, the value of $V(\boldsymbol{N}(x))$ or that of $W(\boldsymbol{N}(x))$ is, so to speak, "standard" average of $\omega(n)$ or that of $\{\Omega(n)-\omega(n)\}$. As is well known ([1: THEOREM 430]):

$$V(\boldsymbol{N}(x))=\log_2 x+A+O\left(\frac{1}{\log x}\right)\,, \tag{1.1}$$

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