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The Structure of Endomorphism Algebras*

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Introduction

Let k be a field and A be an algebra over k with a unity element 1. We denote by $M(A)$ the category of left A-modules. Let Y be an A-module and $E=\mathrm{End}_A(Y)$. We write $M(E)$ for the category of left E-modules and $M'(E)$ for the category of right E-modules.

In this paper we introduce and study an idea of distinguishable modules, which appears quite often in the representation theory of finite groups, by making use of a contravariant representation functor Ψ of $M(A)$ into $M(E)$ (see § 1) and a covariant representation functor Φ of $M(A)$ into $M'(E)$ (see § 3).

DEFINITION (see Definition (2.1)). Assume that an A-module Y is decomposed into a finite number of indecomposable components, say

$$Y=Y_1\oplus Y_2\oplus\cdots\oplus Y_r\ ,$$

and the left A-submodules of soc Y satisfy the D.C.C. Then an indecomposable component Y_ρ, where $1\leqq\rho\leqq r$, is said to be distinguishable (by socle) if soc Y_ρ is multiplicity free and $Y_\rho\cong Y_\sigma$ when soc Y_ρ and soc Y_σ have a same simple submodule up to isomorphism, for any $1\leqq\sigma\leqq r$. When all the indecomposable components $Y_{\rho's}$ are distinguishable, we say that Y has a distinguishable decomposition $Y=Y_1\oplus Y_2\oplus\cdots\oplus Y_r$.

For example when the submodules of Y satisfy the D.C.C. and soc Y is multiplicity free, then Y has a distinguishable decomposition (see [1, Corollary 6.11], [4], [5, Theorem 3.17] and [6, Proposition 2.8 and Corollary 3.5]).

Our main result is as follows

Theorem (see Theorem (2.7)): Let E, Ψ be as above. Assume that

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