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On Projective Normality of Space Curves on a Non-Singular Cubic Surface in P^3

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Introduction

The purpose of this paper is to give a necessary and sufficient condition for space curves on a non-singular cubic surface in P^3 to be arithmetically Cohen-Macaulay. It is known that arithmetically Cohen-Macaulay curves form a smooth open subset in the Hilbert scheme $\mathbf{Hilb}_{P^3}^{p(z)}$ parametrizing curves in P^3 ([2], Théorèm 2). Also the dimension of the Hilbert scheme at a point corresponding to such a curve is calculated in [2], using the free resolution of the curve. There are essentially twelve types of arithmetically Cohen-Macaulay curves on a non-singular cubic surface in P^3. We shall prove this in § 2 and § 3. In § 4, we shall determine free resolutions of these curves. We can determine the arithmetic genus and the degree of the curve by the free resolution.

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§ 1. Statement of the Result.

Let X be a non-singular cubic surface in the projective 3-space P^3 over an algebraically closed field of arbitrary characteristic. Then X is obtained from P^2 by blowing-up six points $P_1, \cdots, P_6$ which are not on a conic and no three of which are collinear. We denote by E_i the exceptional curve corresponding to P_i $(i=1, \cdots, 6)$, and L the total transform of a line in P^2. Then $\mathrm{Pic}\, X \cong Z^7$, with free basis $[L], [E_1], \cdots, [E_6]$ where $[L]$, $[E_i]$ are the linear equivalence class of L, E_i respectively, with intersection numbers

$$\begin{cases} E_i \cdot E_j = -\delta_{ij} & 1 \leqq i, j \leqq 6 \\ L^2 = 1 \\ L \cdot E_i = 0 & 1 \leqq i \leqq 6 . \end{cases}$$

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