TOKYO J. MATH.
VOL. 4, NO. 2, 1981

Normal Gorenstein Surfaces with Ample Anti-canonical Divisor

Fumio HIDAKA and Kei-ichi WATANABE

Waseda University and Tokyo Metropolitan University

Introduction

Let k be an algebraically closed field of arbitrary characteristic and X be a normal projective surface over k. Since every normal surface is Cohen-Macaulay, there exists the dualizing sheaf ω_X on X and X is a Gorenstein surface if and only if ω_X is invertible. (We use the words "dualizing sheaf" and "canonical divisor" interchangeably.) A typical example of a Gorenstein surface is an effective divisor on a non-singular threefold. In this article, we determine the structure of normal Gorenstein surfaces with ample anti-canonical sheaf ω_X^{-1}. If X is non-singular, such a surface is called a Del Pezzo surface and the structure of Del Pezzo surfaces is fairly well-known.

Our first result is that such a surface is either rational or a cone over an elliptic curve and that the singularities on such a surface are rational double points or the unique simple elliptic singular point, according as X is rational or is a cone over an elliptic curve. These results are proved in § 2. In the case k is the complex number field, the same result is obtained by Brenton in [2] using topological properties of ruled surfaces. Our proof uses the theory of resolution of normal surface singularities. An advantage of our proof is that we can treat the problem independent of the characteristic of the base field k.

In § 3, we will study more closely the case that X is rational. We will show that the minimal resolution $\tilde{X}$ of such a surface X can be obtained from $\boldsymbol{P}^2$ by blowing up the points in "almost general position", except for the case when X is a normal quadric surface in $\boldsymbol{P}^3$. Such surfaces are the ones studied by Demazure in [4].

In § 4, we will study the anti-canonical model and the configuration of the singular points on such a surface. If we put $d=\deg X=\omega_X\cdot\omega_X$, we can show that X is a subvariety of degree d in $\boldsymbol{P}^d$ if $d\geqq 3$, a hyper-

Received July 16, 1980