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On Framed Link Groups

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Introduction

In this paper we shall study an invariant of framed links in 3-manifolds and apply it to investigate 3- and 4-manifolds.

In recent years relations between framed links and 3- or 4-manifolds have been revealed by Kirby [3], Montesinos [4] and others. In their papers it was proved that closed orientable 3-(resp. 4-)manifolds correspond to some equivalence classes of framed links in S^3 (resp. $\#_n S^1 \times S^2$) bijectively (resp. injectively). Thus link-theoretical invariants of the equivalence relation can be regarded as invariants of 3-(or 4-)manifolds.

To find such invariants is an important and interesting problem as is mentioned in [2]. But almost all invariants of links known before, for examples, link groups, signatures and so on, are not efficient since they are not invariant under "band moves". On the contrary, framed link groups which we shall define in the following section are invariant under these moves. Moreover they change in a simple fashion when they are "stabilized".

In § 1 we shall give the definition of framed link groups and prove its invariance under "band moves". In § 2 and § 3 we shall study the relations between framed link groups and 3- and 4-manifold theories.

§ 1. Framed link groups.

We work in the smooth category. Let M^3 be a connected 3-manifold without boundary.

DEFINITION 1. $L \subset M$ is called a framed link in M with k components if L is a disjoint union of circles l_i $(i=1, 2, \cdots, k)$ with tubular neighbourhoods N_i and framings $f_i: l_i \times D^2 \to N_i$ such that $N_i \cap N_j = \emptyset$ $(i \neq j)$.

REMARK. For a link in a 3-sphere S^3 or a 3-disk D^3, a framing is indicated by an integer as in [3].

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