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Totally Real Parallel Submanifolds in $P^n(c)$

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Introduction

It is an interesting problem to classify the parallel submanifolds in a specific riemannian symmetric space. Actually, these submanifolds have been classified by D. Ferus [5], [6], [7] when the ambient space is the Euclidean space or the Euclidean sphere, and by M. Takeuchi [17] when the ambient space is the real hyperbolic space. Moreover H. Nakagawa and R. Takagi [10] and M. Takeuchi [16] have classified the parallel Kähler submanifolds in the complex projective space $P^n(c)$ with constant holomorphic sectional curvature c. It is known that parallel non-Kähler submanifolds in $P^n(c)$ are totally real.

In this paper we study n-dimensional complete totally real parallel submanifolds in $P^n(c)$. It is known that a riemannian manifold which admits a parallel isometric immersion into a riemannian symmetric space is a locally symmetric space. Fix an n-dimensional simply connected riemannian symmetric space M^n. Let $\bar{\mathscr{T}}_M$ (resp. $\bar{\mathscr{S}}_M$) be the set of all equivalence classes of totally real parallel isometric immersions of M^n into $P^n(c)$ (resp. of complete totally real parallel submanifolds in $P^n(c)$ with the universal riemannian covering M^n). Moreover, in section 3 we define an equivalence relation among symmetric trilinear forms on a tangent space of M satisfying certain conditions, and denote by $\bar{\mathscr{M}}_M$ the set of all equivalence classes of these trilinear forms. In sections 2, 3, we shall show that there are the natural correspondences among these sets $\bar{\mathscr{T}}_M$, $\bar{\mathscr{S}}_M$, $\bar{\mathscr{M}}_M$. In sections 4, 5, we shall determine the set $\bar{\mathscr{M}}_M$ for a riemannian symmetric space M without Euclidean factor. Moreover, in section 6, we shall study the set $\bar{\mathscr{M}}_M$ for a riemannian symmetric space M with Euclidean factor and an interesting example in the geometry of totally real surfaces in $P^2(c)$.

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