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On Regular Fréchet-Lie Groups III

A Second Cohomology Class Related to the Lie Algebra of Pseudo-Differential Operators of Order One

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Introduction

Fourier integral operators have been defined by Hörmander [5], and developed extensively by himself and many other authors as a tool of studying fundamental solutions of Cauchy problems of pseudo-differential equations of hyperbolic type. However, if we deal with a Fourier integral operator F defined on a manifold, we see immediately that the expression of F contains usually a huge ambiguity. Phase functions and amplitude functions do not have invariant meanings under the change of local coordinate systems, and the rule of coordinate transformations is usually a very complicated one. Therefore, there arise several difficulties to define a topology, for instance, on the space $\mathscr{F}^0$ of all Fourier integral operators of order 0.

In [11], we gave a sort of global expression of Fourier integral operators and in [12] we defined a "vicinity" $\mathfrak{N}$ of the identity operator in the space $\mathscr{F}^0$ such that $\mathfrak{N}$ satisfies the properties of a topological local group. Moreover we have shown in [11] that $F\in\mathfrak{N}$ can be expressed in an "almost" unique fashion, if we fix a C^∞ riemannian metric on N.

Let us explain this situation at first. Let $\mathscr{D}_\Omega^{(1)}$ be the group of all symplectic transformations of order one on $T^*N-\{0\}$, where T^*N is the cotangent bundle a closed C^∞ riemannian manifold N. It is known that $\mathscr{D}_\Omega^{(1)}$ is isomorphic to the group $\mathscr{D}_\omega(S^*N)$ of all contact transformations on the unit cosphere bundle S^*N. Since $\mathscr{D}_\omega(S^*N)$ is a topological group under the C^∞ topology, we give the same topology on $\mathscr{D}_\Omega^{(1)}$ through

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