TOKYO J. MATH.
VOL. 4, NO. 2, 1981

On Regular Fréchet-Lie Groups II

Composition Rules of Fourier-Integral Operators on a Riemannian Manifold

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Introduction

In the previous paper [8], we gave a differential geometrical expression of Fourier-integral operators on a closed riemannian manifold N without using local coordinate pathches, which is expressed in the following relatively concrete form: (Cf. (19) for the precise meaning of the notations.)

$$(1)\qquad \begin{aligned}&(Fu)(x)\\ &\quad=\sum_{\alpha}\iint \lambda_\alpha a(x;\xi,X)e^{-i\langle\varphi_2(x;\xi)|X\rangle-i|\xi|A_\alpha(X)}(\nu u)^{\cdot}(\varphi_1(x;\xi);X)\,đX\,đ\xi\\ &\quad\quad+(K\circ u)(x)\ ,\end{aligned}$$

where $\varphi=(\varphi_1;\varphi_2)$ is a symplectic transformation of order 1 on $T^*N-\{0\}$. Although our operators such as (1) form much narrower class than what was defined by Hörmander [3] or Guillemin -Sternberg [2], our expression contains less ambiguities, and hence one can give a sort of coordinate system on a "vicinity" of the identity operator of the Fourier-integral operators of order 0 (cf. Theorem 5.8 [8]). Moreover, the above expression seems to be convenient for concrete computation of the fundamental solution of the equation

$$(2)\qquad \frac{d}{dt}u=\sqrt{-1}\,Pu$$

for a pseudo-differential operator P of order 1 with a real principal symbol. We shall state the reason in what follows.

Let $G\mathscr{F}^0$ be the group generated by the invertible Fourier-integral operators of order 0, written in the form (1). We regard $G\mathscr{F}^0$ as if it

Received July 18, 1980