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Vanishing Theorems of Cohomology Groups with Values in the Sheaves $\mathcal{O}_{\mathrm{inc},\varphi}$ and $\mathcal{O}_{\mathrm{dec}}$

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Introduction

In this paper we study vanishing theorems of cohomology groups with values in the sheaves of holomorphic functions with exponential bounds. We treat the sheaf $\mathcal{O}_{\mathrm{inc},\varphi}$ of holomorphic functions with some exponential growth condition and the sheaf $\mathcal{O}_{\mathrm{dec}}$ of holomorphic functions with some exponential decay condition. The sheaves $\mathcal{O}_{\mathrm{inc},\varphi}$ and $\mathcal{O}_{\mathrm{dec}}$ are proposed by Professors M. Sato and T. Kawai to define modified Fourier hyperfunctions. Those are modifications of the sheaf $\tilde{\mathcal{O}}$ of holomorphic functions with the infra-exponential growth condition and the sheaf $\underset{\sim}{\mathcal{O}}$ of holomorphic functions with some exponential decay condition in Kawai [12]. We have two motivations:

The first is to give a foundation for our forthcoming paper (Saburi [26]) on the theory of modified Fourier hyperfunctions.

The second is to improve Kawai's proof of vanishing theorems of cohomology groups with the value in the sheaf $\tilde{\mathcal{O}}$. Our methods of proof are valid for the $\tilde{\mathcal{O}}$ without difficulties.

Kawai proved the Cartan Theorem B and the Malgrange theorem for the sheaf $\tilde{\mathcal{O}}$ (Theorems 2.1.4 and 3.1.8 in Kawai [12] respectively). His proof of the Cartan Theorem B for the sheaf $\tilde{\mathcal{O}}$ is somewhat complicated. Moreover it seems to the author that his proof of the Malgrange theorem for the sheaf $\tilde{\mathcal{O}}$ is not complete.

We give a direct method of the calculation of the cohomology groups with the value in the sheaf $\mathcal{O}_{\mathrm{inc},\varphi}$, and prove the Cartan Theorem B for that sheaf (Theorem I in §1.2). We also prove in details the Malgrange theorem for the sheaf $\mathcal{O}_{\mathrm{inc},\varphi}$ (Theorem III in §1.2).

There are some works relevant to Kawai [12]. Those are Ito-Nagamachi

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