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Asymptotic Strong Convergence of Nonlinear Contraction Semigroups

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Introduction

Let $S=\{S(t): t\geqq 0\}$ be a (nonlinear) contraction semigroup on a closed convex subset C of a Hilbert space H. In this note we study the asymptotic strong convergence of the orbits $S(t)x$ $(x\in C)$ of S. In 1975 Bruck [5] discussed this problem for a nonlinear contraction semigroup S under the assumption that S is generated by the subdifferential $\partial\varphi$ of a proper lower semicontinuous convex functional φ, and that φ is even in the sense that $\varphi(x)=\varphi(-x)$ on its effective domain $\mathfrak{D}(\varphi)=\{x\in H: \varphi(x)<+\infty\}$. Since then a number of extended forms of Bruck's conditions for the asymptotic strong convergence have been obtained, for instance, in the works of [1], [4], [7], [8] and [10]. Here some other sufficient conditions on the generator A of S for the existence of strong limits of Cèsaro means $(1/t)\int_0^t S(\tau+h)xd\tau$ as well as those of orbits $S(t)x$ are investigated.

The present paper contains three results. The first result (Theorem 1) provides a sufficient condition for the strong convergence of the orbit of S generated by the subdifferential of a proper lower semicontinuous functional φ. This result extends the author's previous result in [10] and so involves the case in which φ is even. On the other hand, if S is generated by the subdifferential of φ which assumes a minimum in H and if there exists a real number $\lambda>\min\varphi$ such that the set $M_\mu=\{x\in D(\partial\varphi): \varphi(x)\leqq\lambda$ and $\|x\|\leqq\mu\}$ is relatively compact for each $\mu>0$, then it is proved that $S(t)x$ converges strongly to a minimum point of φ as $t\to\infty$. Our result involves this case as well. It turns out that Theorem 1 extends the above-mentioned two results which are of completely different types. The second result (Theorem 2) is concerned with the asymptotic strong convergence of the Cèsaro means of S as well as the orbits of S them-

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