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On Closed Subalgebras Lying Between A and H^∞, II

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Introduction

Let D be the open unit disc in the complex plane C, $\bar{D}$ its closure and T its boundary, the unit circle. The basic algebras appearing in this paper are the algebra $C(T)$ of continuous functions on T, and the algebra L^∞ of essentially bounded, measurable functions with respect to the normalized Lebesgue measure $d\theta/2\pi$ on T. These are Banach algebras under the supremum and essential supremum norms, respectively. We denote by A and H^∞ the closed subalgebras of $C(T)$ and L^∞, respectively, whose Fourier coefficients with negative indices vanish. Let $M(B)$ be the maximal ideal space of a uniform algebra B, and $\partial(B)$ be the Silov-boundary. We recall that L^∞ is isometrically isomorphic to $C(M(L^\infty))$ and that $\partial(H^\infty)=M(L^\infty)$. Let us denote $M(L^\infty)$ by X.

The closed subalgebras of L^∞, called Douglas algebras, which contain H^∞ properly, are studied in connection with Toeplitz operators. There is a smallest such algebra, namely, the closed subalgebra of L^∞ generated by H^∞ and $C(T)$, which is denoted by $[H^\infty, C(T)]$. This algebra turns out to be equal to $H^\infty+C(T)$, the linear span of H^∞ and $C(T)$ ([14]). A. Chang [3] and D. E. Marshall [10] showed that every Douglas algebra B is characterized as an algebra $B=H^\infty+C_B$, where C_B is the C^*-algebra generated by the inner functions invertible in B. This fact inspired our interest in characterizing or classifying closed subalgebras of H^∞ containing A, which will be called hereafter analytic subalgebras in this paper. The algebras $H^\infty\cap C_B$, defined in [4] is only well-known class of analytic subalgebras, which are associated with the Douglas algebras $H^\infty+C_B$. The first thing for this purpose, we shall construct in § 1 a new class of analytic subalgebras and seek common properties for these algebras, hopefully, for all the analytic subalgebras. Second, we shall be concerned in § 2 and § 4 with two problems, which will be mentioned

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