

Uniqueness for the Characteristic Cauchy Problem and its Applications

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Introduction

In this paper we will consider the local uniqueness for Fuchsian partial differential operators (See [2]) with C^∞ -coefficients and as its applications we shall give some examples in the case of partial differential operators with characteristic or non-characteristic initial surfaces.

The local uniqueness for a Fuchsian partial differential operator has been obtained by Baouendi and Goulaouic [2], when its operator has analytic coefficients with respect to space variables x . Recently Alinhac and Baouendi [1] studied this problem for some characteristic pseudo-differential operators on a compact manifold. For other many works for characteristic operators, we wish the reader to consult references of [1] and [2].

On the other hand in the case of a non-characteristic initial surface there is well-known Holmgren's theorem for a differential operator with analytic coefficients. Calderón [3] showed the local uniqueness result for non-characteristic partial differential operators with non-analytic coefficients, assuming that coefficients of a principal symbol are real-valued and its characteristic roots are simple from each other. When characteristic roots have multiplicity, many works are found in Hörmander [4], Mizohata [6], Matsumoto [5], Watanabe [11], Zeman [13] and others. In the above referred papers, all the authors assume that a imaginary part of each characteristic root never vanishes or vanishes identically. When this assumption is not satisfied, Kumano-go [12], Nirenberg [7] studied some partial differential operators and recently Strauss and Trèves [8] considered a first order partial differential operator.

The aim of this paper is to show that for some differential operators with C^∞ -coefficients we can treat the local uniqueness for a characteristic Cauchy problem and non-characteristic one in the same frame. In our