TOKYO J. MATH.
VOL. 5, NO. 1, 1982

On an Analogoue to Hecke Correspondence

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Introduction

In [2], Hecke established the one-to-one correspondence, called Hecke correspondence, between (A) and (B) below, through Mellin and inverse Mellin transformation: (k: even, $k\geqq 4$)

(A) (i) $f(z)$ is analytic on the upper-half plane H,

(ii) $f(\sigma(z))=(cz+d)^k f(z)$ for $\sigma=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\in\Gamma$ (Γ: the full modular group),

(iii) $f(z)=\sum_{n=0}^{\infty} a(n)e^{2\pi inz}$.

(B) (i) If $a(0)=0$, then

$$\varphi(s)=\sum_{n=1}^{\infty}\frac{a(n)}{n^s}$$

is continued to an entire function of s.

(ii) If $a(0)\neq 0$, then $\varphi(s)$ is continued to s-plane analytically except for a simple pole at $s=k$ with residue

$$\frac{(-1)^{k/2}a(0)(2\pi)^k}{\Gamma(k)}.$$

(iii) $(2\pi)^{-s}\Gamma(s)\varphi(s)=(-1)^{k/2}(2\pi)^{s-k}\Gamma(k-s)\varphi(k-s)$.

This is a vast generalization of Hamburger's Theorem on the determination of Riemann zeta-function via the functional equation.

Note that in (A), it follows that

$$\text{if}\quad a(0)=0\quad\text{then}\quad a(n)=O(n^{k/2})$$

and

$$\text{if}\quad a(0)\neq 0\quad\text{then}\quad a(n)=O(n^{k-1}).$$

Now we shall consider

Received March 16, 1981